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CHAOTIC RESPONSE OF A COMPOSITE PLATE

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Abstract—The chaotic response of a six-layered anti-symmetric cross-ply $(0°/90°)_3$ rectangular plate with immovable edges is studied. The plate is assumed to be damped and acted upon by a harmonic force $f_0 \sin T$. For a value of $f_0 = 97$, it is shown that chaos occurs.

1. EQUATION OF MOTION

Consider a thin rectangular plate of total thickness, t, composed of many orthotropic layers, layered at 0° and 90° alternately, as shown in Fig. 1. For large amplitude motion, Singh *et al.* [1] have shown using the general theory of anti-symmetric cross-ply plates based on Kirchoff's hypothesis with the incorporation of Karmen-type strain–displacement relations, that modal equation can be written as

$$(\Sigma\, \rho_i t_i)\frac{\mathrm{d}^2 w}{\mathrm{d}T^2} + \alpha w + \beta w^2 + \gamma w^3 = 0. \qquad (1)$$

That is

$$\frac{\mathrm{d}^2 w}{\mathrm{d}T^2} + \omega_L^2\left[w + \left(\frac{\beta}{\alpha}\right)w^2 + \left(\frac{\gamma}{\alpha}\right)w^3\right] = 0. \qquad (2)$$

For a six-layered, anti-symmetric cross-ply $(0°/90°)_3$ rectangular plate of immovable edges and size $a = 20$ mm; $b = 10$ mm; $t_i = 0.1$ mm and total thickness $t = \Sigma\, t_i = 0.6$ mm; these coefficients have been reported by Pillai and Nageswara Rao [2] for the mode $m = 1$, $n = 1$. They are $\alpha = 0.5652$; $\beta = 0.5028$; $\gamma = 3.3433$. These coefficients are reported for elastic constants $E_{11} = 5000$ kg mm^{-2}; $E_{22} = 500$ kg mm^{-2}; $\nu_{12} = 0.25$ and $G_{12} = 250$ kg mm^{-1}.

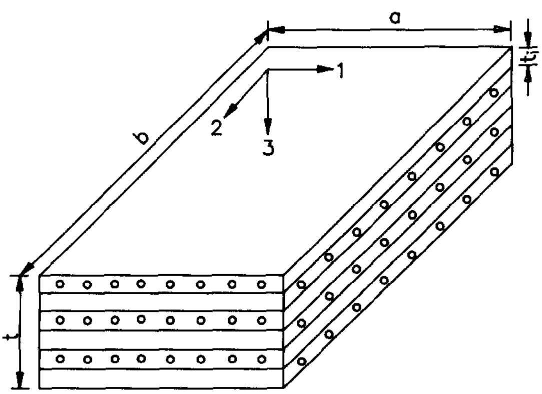

Fig. 1. Geometry of a cross-ply rectangular plate.

Assuming the plate to be viscous-damped and acted upon by a harmonic force, the equation of motion can be written as

$$\frac{\mathrm{d}^2 w}{\mathrm{d}T^2} + 0.1\frac{\mathrm{d}w}{\mathrm{d}T} + w + 0.8896w^2 + 5.9153w^3 = f_0 \sin T. \qquad (3)$$

2. SOLUTION OF THE EQUATION OF MOTION

Transposing the non-linear terms to the right-hand side as an equivalent load, eqn (3) can be written as

$$\frac{\mathrm{d}^2 w}{\mathrm{d}T^2} + 0.1\frac{\mathrm{d}w}{\mathrm{d}t} + w = f_0 \sin T - (0.8896w^2 + 5.9153w^3). \qquad (4)$$

This has been solved using the Newmark method (trapezoidal rule) with equilibrium iteration and variable time stepping based on convergence criteria. In addition, the temporal errors are contained by limiting the half step residual to within 1%. The details of the algorithm and the solution scheme have been reported earlier by the authors [3].

3. RESULTS

Figure 2 shows the Poincare plot for a value of $f_0 = 96$. It can be seen that it is a period – 2 motion. Figure 3 shows the Poincare plot for a value of $f_0 = 97$. Figure 4 shows the time history of the above response and Fig. 5 the phase plot.

It can be concluded that chaos occurs in the composite plate at $f_0 = 97$.

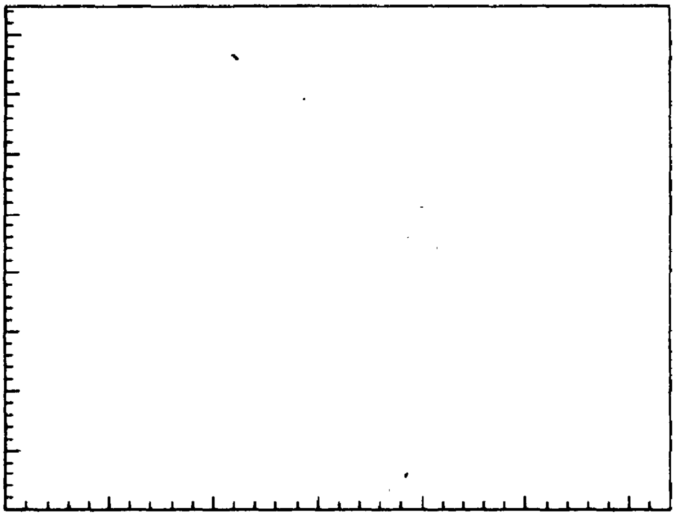

Fig. 2. Poincare plot of composite plate at excitation, $f_0 = 96.0$.

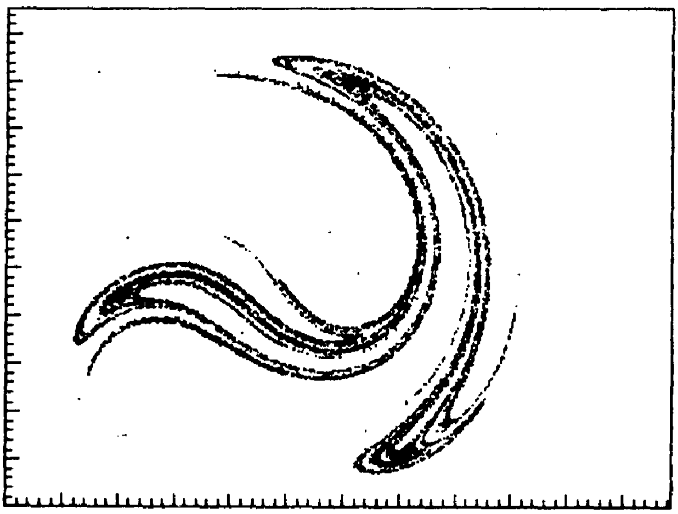

Fig. 3. Poincare plot of composite plate at excitation, $f_0 = 97.0$.

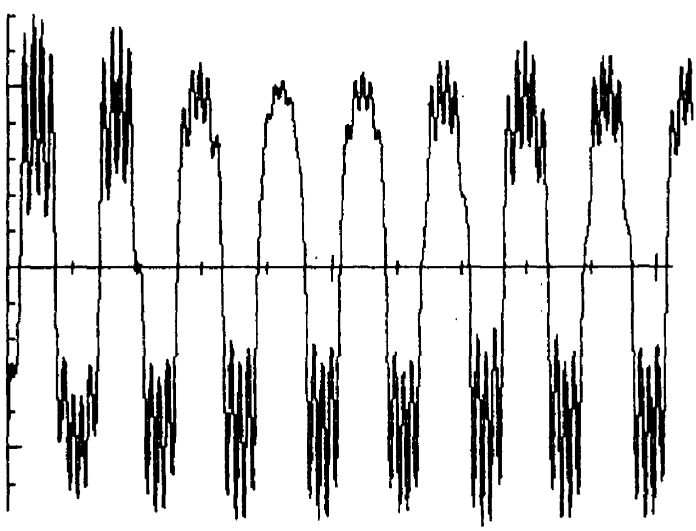

Fig. 4. Time history of composite plate at excitation, $f_0 = 97$.

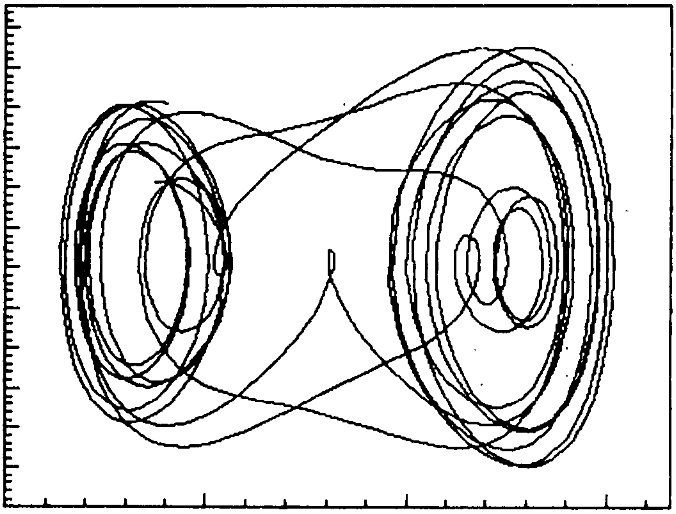

Fig. 5. Phase plot of composite plate at excitation, $f_0 = 97$.

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