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Limit load analysis and safety assessment of an elbow with a circumferential crack under a bending moment

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The use of the leak-before-break concept in the design of a Primary Heat Transport (PHT) piping system for nuclear power plants requires postulation of the largest credible flaws at highly stressed points and demonstration of system stability under the most severe loading conditions. In the PHT piping system, the elbows and branch tees are normally found to be among the most highly stressed piping components. This necessitates detailed flaw evaluation of these components. In the present paper, safety assessment of a pump discharge elbow of 500 MWe Indian PHWR with a throughwall circumferential crack under a bending moment is carried out using the R6 approach. The stress intensity factors (SIF) have been computed by using a database of SIF for elbows with cracks which was generated in one of our recent studies. Limit loads at plastic collapse have been evaluated by carrying out non-linear finite element analysis. Finally, the safety assessment of the pump discharge elbow is performed by doing a sensitivity study on crack length, applied moment and material fracture toughness.

NOTATION

a	Crack length
A_e	A non-dimensional parameter to express SIF
C	A constant to express the normalised limit moment of an elbow with a crack
C_n	Coefficients to express A_e
D	Mean diameter of the pipe
E	Young's modulus
h	Pipe factor (tR/r^2)
K_{mid}	SIF at mid-layer
M	Applied moment
M_l	Limit moment
M_0	Limit moment without any crack
n	Strain-hardening exponent in Ramberg–Osgood relationship
p	Internal pressure
p_n	Exponents to express A_e
r	Mean pipe radius
R	Mean pipe bend radius
t	Pipe thickness
α	Material constant in Ramberg–Osgood relationship
θ	Angle subtended by the elbow axis at the bend centre
λ	Longitudinal crack in non-dimensional form $(a/\sqrt{(rt)})$
ν	Poisson's ratio
ρ	Load ratio to define the relative magnitude of internal pressure and bending moment
σ_f	Flow stress, $(\sigma_u + \sigma_y)/2$
σ_r	Reference stress to define A_e
σ_u	Ultimate stress
σ_y	Yield stress
ϕ	End-point rotation of elbow
ψ	Angle subtended by the elbow circumference at the cross-sectional centre at a fixed θ (measured from intrados)
ψ_c	Semi-circumferential crack angle

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1 INTRODUCTION

The concept of Leak-Before-Break (LBB) has replaced the traditional design basis event of Double Ended Guillotine Break (DEGB) in the design of Primary Heat Transport (PHT) piping systems for nuclear power plants. The use of the LBB concept in the design of a piping system requires the postulation of the largest credible flaws at highly stressed points, and demonstration of system stability under the most severe loading conditions. Stress analysis of a PHT piping system shows that the most highly stressed piping components are normally elbows and branch tees. This necessitates detailed flaw evaluation of different piping components such as elbows and tees. While there are a number of methods to assess the integrity of structures containing defects, the R6 method is one of the most widely used. It assesses the safety of a flawed structure with respect to the two limiting conditions of structural failure, namely brittle fracture and plastic collapse. Safety of the structure is ensured if the assessment points lie within the area bounded by the axis of the failure assessment diagram and the failure assessment line.

There are four inputs required to evaluate the assessment points for a structure: the stress intensity factor (SIF), material fracture toughness, the applied load and the limit load of the structure at plastic collapse. Out of these four, fracture toughness is a material property and the applied load is known from the design analysis. However the other two parameters, i.e. SIF and limit load at plastic collapse, are geometry dependent, and also vary with crack size. While the equations of SIF and limit load are well established for simpler geometries, e.g. cracked plate or straight pipe, the same is not true for more complicated geometries such as elbow and branch tees. Some versatile numerical technique such as the finite element technique should be used to calculate limit load and SIF for these geometries.

In the present work a safety assessment of a pump discharge elbow of 500 MWe Indian PHWR with a circumferential crack under a bending moment is carried out using the R6 approach. Stress intensity factors have been calculated from a database of SIF for various sizes of elbow with longitudinal and circumferential cracks which was generated in one of our recent works.[1] The limit loads have been determined by carrying out elasto-plastic finite element analysis of an elbow with circumferential cracks. These have been used as inputs to carry out the safety assessment of the given elbow.

2 DEVELOPMENT WORK

2.1 Limit load computation of an elbow

Limit load solutions for an elbow under a bending moment are not very well established in the literature. While reviewing the test results for the limit load of an elbow,[2] Miller cautions the user of these results because of their limited validation. Hence in the present work it was decided to carry out elasto-plastic finite element analysis to evaluate the limit bending moment at plastic collapse of an elbow with a circumferential crack under in-plane bending. The collapse load is determined from the load–deflection curve by the twice elastic slope method. In elasto-plastic analysis, material hardening is considered in order to calculate a realistic load carrying capacity for the structure.

Description of the finite element code 'FABS'

A finite element code 'FABS' (Fracture mechanics Analysis of Bending Structure) is used to determine the non-linear moment rotation curve of the elbow. The code uses eight-noded degenerate shell bending elements. It uses von Mises yield criteria with associated Prandtl–Reuss flow theory. Plastic flow along the thickness is considered using a layered approach. Each layer contains stress points on its mid-surface. The stress components of the layer are computed at these stress points and are assumed to be constant over the thickness of each layer. Thus, the actual stress distribution of the shell is modelled by a piecewise constant approximation. Besides the mechanical load, the code considers the thermal load due to changes in temperature, the temperature gradient and the rate of change of mechanical properties with temperature. The modified Newton–Raphson method is used to solve the equations of plasticity in an iterative manner.

A case study

Though many benchmark case studies have been performed to validate the 'FABS' code, the computed results for the collapse analysis of an elbow under a bending moment as described in Ref. 3 are shown here. The purpose of this study is to check the accuracy and adequacy of the present finite element code in modelling the collapse behaviour of an elbow. This has been done by comparing the computed moment rotation curve with those quoted in Ref. 3.

The elbow considered in Ref. 3 has been analysed in the present case by modelling half of the geometry using the plane of symmetry. There are 315 eight-noded thick-shell elements and 1018 nodes employed in the modelling. The geometrical dimensions and the material properties are taken as the same as in Ref. 3. We have solved this case by imposing a couple on the thick plate connected at the end of the elbow and, independently, a linearly varying load directly at the end of the elbow. However, both these loading conditions have shown identical results. Figure 1 compares the present moment rotation curve with those quoted in Ref. 3. A good match may be observed.

2.2 Stress intensity factor calculations for elbows with cracks

In one of our recent studies, a database was generated to evaluate stress intensity factors of elbows with throughwall cracks under combined internal pressure and a bending moment. SIF is expressed as

$$A_e = K_{mid}/\sigma_r\sqrt{(\pi a)}; \quad \sigma_r = M/\pi r^2 t + pr/2t$$

Three parameters are chosen to characterise a cracked elbow, namely the pipe factor (h), r/t and the crack length. Another parameter (ρ) is selected to consider the relative magnitude of stresses due to internal pressure and a bending moment. It is expressed as

$$\rho = (2/\pi).(M/pr^3)$$

A_e is expressed in terms of these parameters. The equation used to express A_e for a circumferential crack is given in Ref. 1:

$$A_e = (C_1 + C_2 h^{p_1}) + (C_3 + C_4 h^{p_2}).(\psi_c/\pi)^{p_3} + [(C_5 + C_6 h^{p_4}) + (C_7 + C_8 h^{p_5}).(\psi_c/\pi)^{p_6}].(t/r)^{p_7} \quad (1)$$

For a longitudinal flaw, (ψ_c/π) is replaced by λ in the above expression. The values of the coefficients (C_n) and exponents (p_n) have been derived in Ref. 1 for various ρ. The present case corresponds to $\rho = \alpha$ for a circumferential crack.

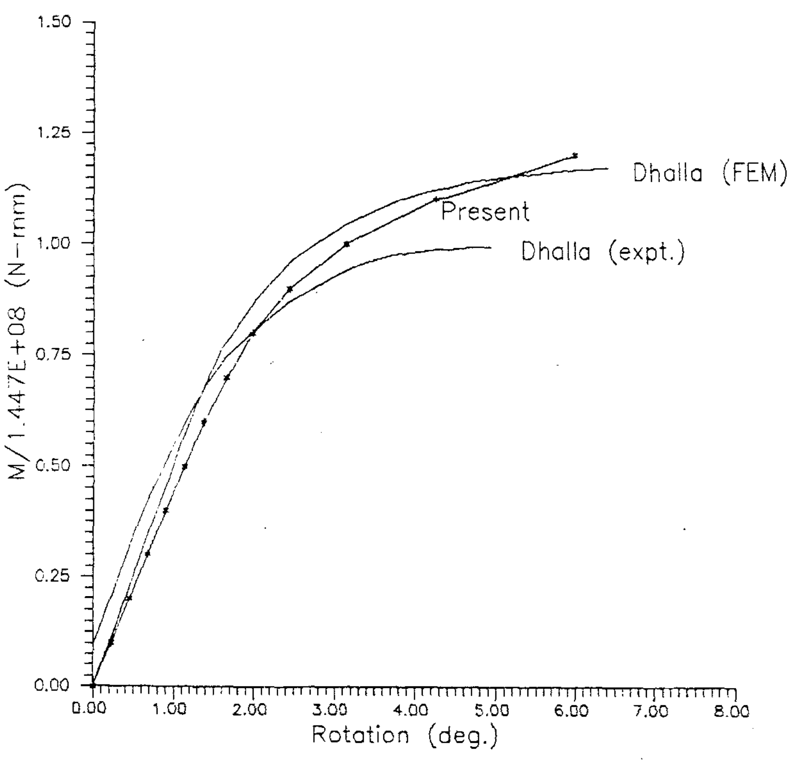

Fig. 1. End plane rotation of an elbow under bending moment compared to Ref. 3.

3 ANALYSIS OF A PUMP DISCHARGE ELBOW

Figure 2 shows the schematic layout of a primary heat transport (PHT) piping system of 500 MWe Indian PHWR. Essentially it consists of a steam generator inlet line, a steam generator outlet line, a pump discharge line, a reactor inlet header and a reactor outlet header. Stress analysis of this PHT piping system shows that elbow and branch tees are normally highly stressed piping components. In the present case, limit load analysis and safety assessment of the pump discharge elbow (Fig. 2) with a postulated circumferential crack under a bending moment is carried out.

Prior to analysing the pump discharge elbow with cracks, the non-weakened elbow is analysed. Figure 3 shows the geometry of the pump discharge elbow. The material of the elbow is SA333 Gr.6. The relevant material properties are

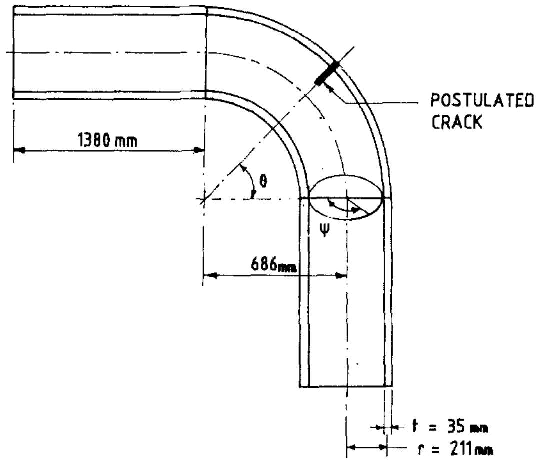

Fig. 3. Geometry and location of postulated circumferential crack on pump discharge elbow.

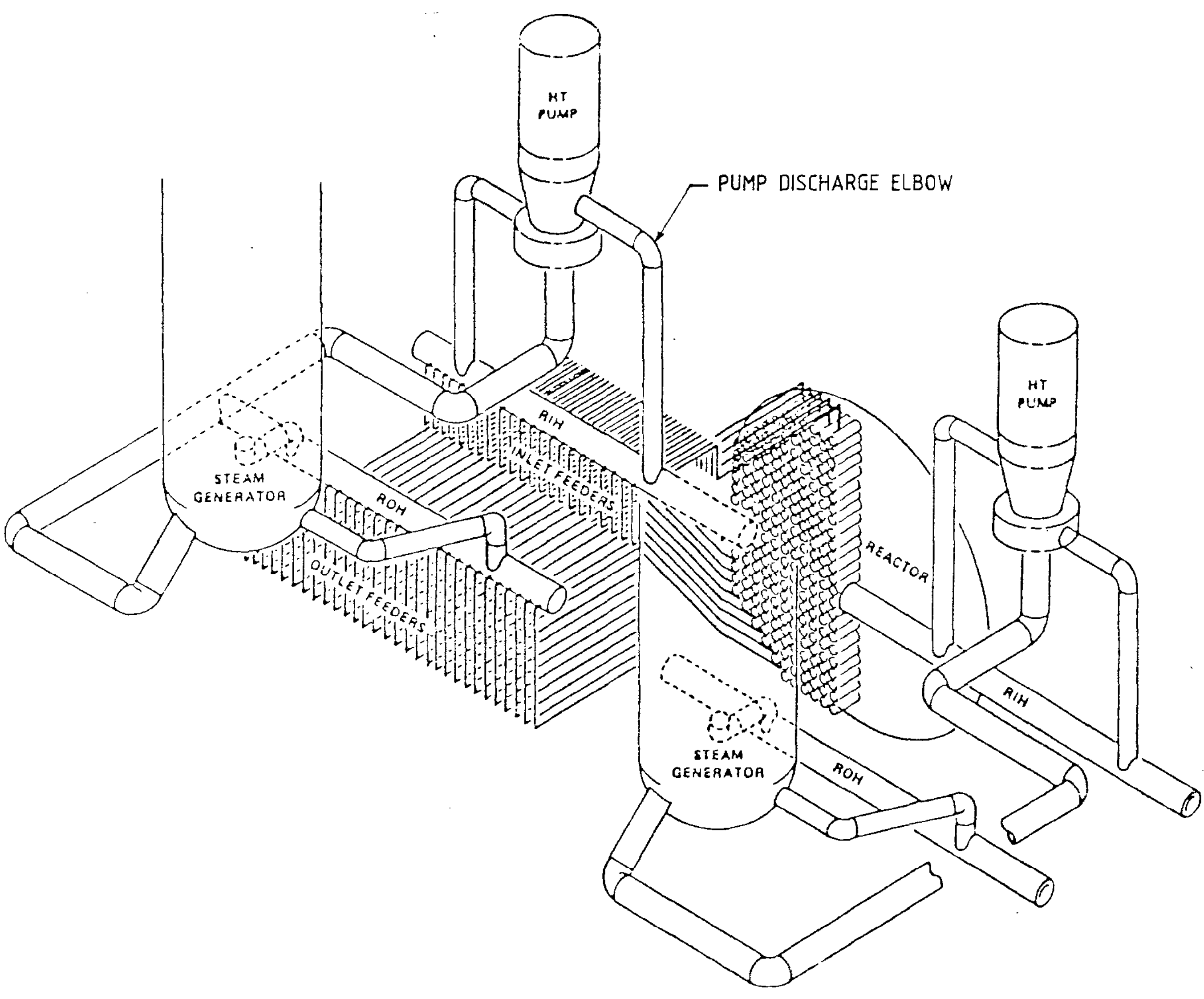

Fig. 2. Schematic layout of 500 MWe Indian PHT piping.

Table 1. Material properties of SA333 Gr.6

$\varepsilon/\varepsilon_0 = \sigma/\sigma_0 + \alpha(\sigma/\sigma_0)^n$
$\alpha = 2{\cdot}51$, $n = 4{\cdot}2$, $E = 179$ GPa
$\sigma_y = 187$ MPa, $\sigma_u = 428$ MPa, $J_{ic} = 105$ kN/m

shown in Table 1.[4] The elbow is modelled with a connecting straight pipe. The length of the straight pipe is chosen as six times the radius of the pipe to eliminate the effect of ovalisation at the point where the load is applied. A moment is applied at the end of the straight pipe as a triangularly varying distributed load. Only one quarter of the elbow is modelled due to symmetry. Figure 4 shows the finite element mesh with the mirror image on the XZ plane. The mesh consists of 225 elements and 736 nodes, and the moment rotation curve of the elbow is subsequently generated. The applied moment is normalised with respect to the collapse moment of a straight pipe under bending ($4r^2t\sigma_f$) based on the material flow stress. Figure 5 shows the moment rotation curve of the elbow considering the actual material stress/strain curve. This figure also shows the limit moment using the relationship suggested in Ref. 2, based on flow stress:

$$M_0/4r^2t\sigma_f = 0{\cdot}94(h)^{2/3} \tag{2}$$

From the figure it may be seen that the asymptotic limit moment using actual material strain hardening is higher than the limit moment predicted by eqn (2).

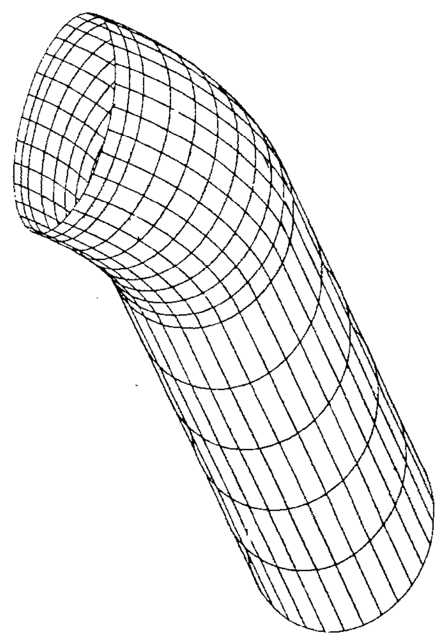

Fig. 4. Finite element mesh of the pump discharge elbow.

4 LIMIT MOMENT COMPUTATION OF PUMP DISCHARGE ELBOW WITH CIRCUMFERENTIAL CRACKS UNDER BENDING

A circumferential crack is postulated at the mid-plane of the pump discharge elbow with its centre at the extrados (Fig. 3). The moment rotation curve is calculated for different crack angles considering the material hardening of

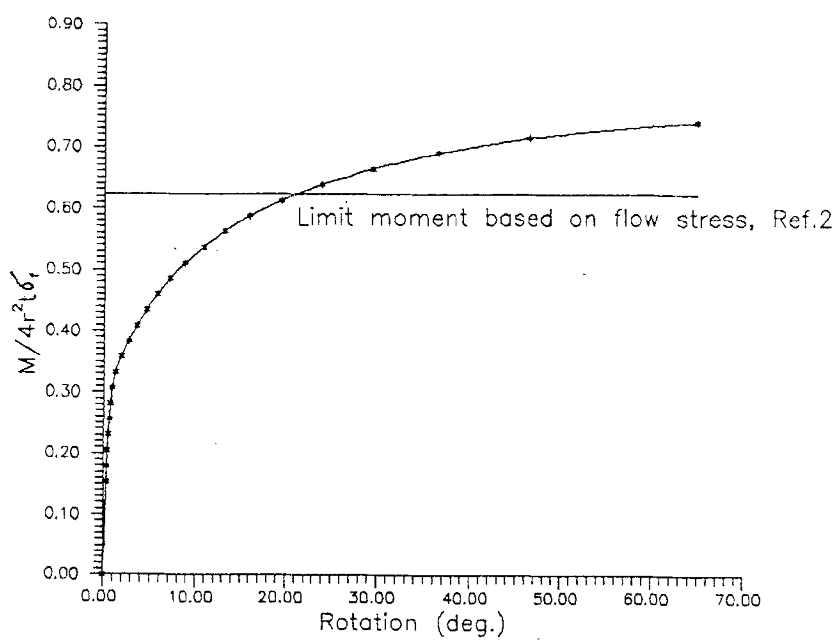

Fig. 5. End plane rotation of pump discharge elbow without crack under bending moment.

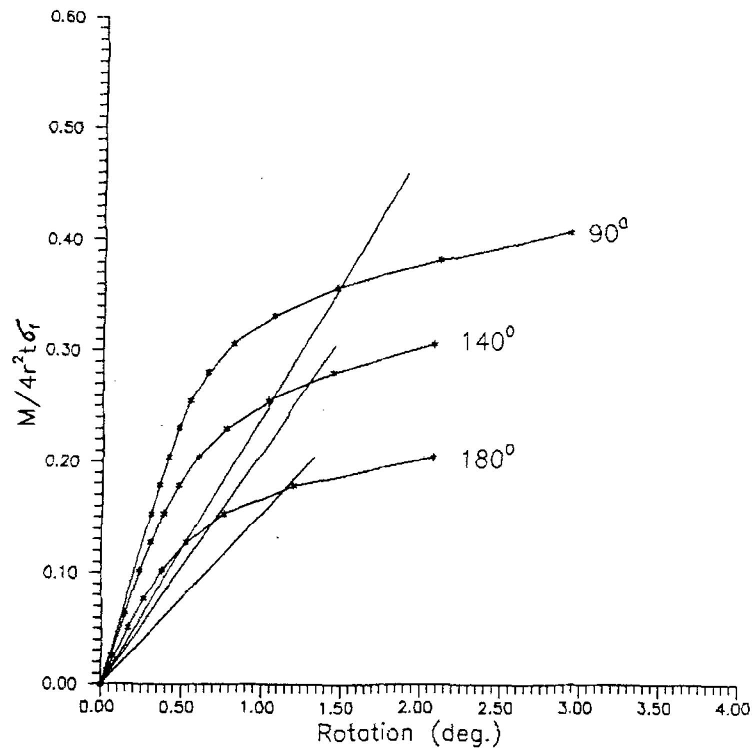

Fig. 6. End plane rotation of pump discharge elbow with various cracks under bending moment.

Table 2. Normalised limit moment of the pump discharge elbow with various circumferential cracks

ψ_c	45°	70°	90°
M_l/M_0	0·578	0·437	0·286

SA333 Gr.6. Figure 6 shows the moment rotation curves for crack angles $2\psi_c = 90°$, 140° and 180°. The gradual decrease in structural stiffness by increasing crack length is denoted by the decrease in the gradient of the moment rotation curve. Collapse moments are then evaluated for different crack angles by the twice elastic slope method. Table 2 shows the limit moment of the pump discharge elbow with various sizes of crack. The limit moments of elbows with cracks are normalised with respect to the limit moment for an uncracked elbow using eqn (2) for the same pipe factor. If an equation of the form

$$M_l/M_0 = 1 - C(\psi_c/\pi) \quad (3)$$

is fitted to the above results, the average value of 'C' works out to 1·52, which is very close to the value of $C = 1{\cdot}5$ suggested in Ref. 2 for a circumferential crack extending from crown to extrados.

5 SAFETY ASSESSMENT OF PUMP DISCHARGE ELBOW BY R6 METHOD

A circumferential crack is postulated at the extrados of the pump discharge elbow (Fig. 2) and its safety is assessed by the R6 method. A material-specific failure assessment diagram (option 2) is chosen here. The failure assessment line is terminated at $(L_r)_{max} = \sigma_f/\sigma_y = 1{\cdot}65$. Moments applied to the elbow due to self weight, thermal expansion and SSE loadings are considered in the analysis. All of the stresses are categorised as primary stresses. Table 3 shows the different components of the moment. Regarding the category of the analysis, category 1 is chosen here.[5] Fracture toughness (J_{ic}) is kept constant. No advantage is taken of the increasing fracture resistance of the material with increasing crack length. Following the category 1 analysis, a

Table 3. Components of moment loading

M (kN + m)	Self weight	Thermal expansion	SSE
M_x	20	108	20
M_y	−20	−96	21
M_z	−4	112	42

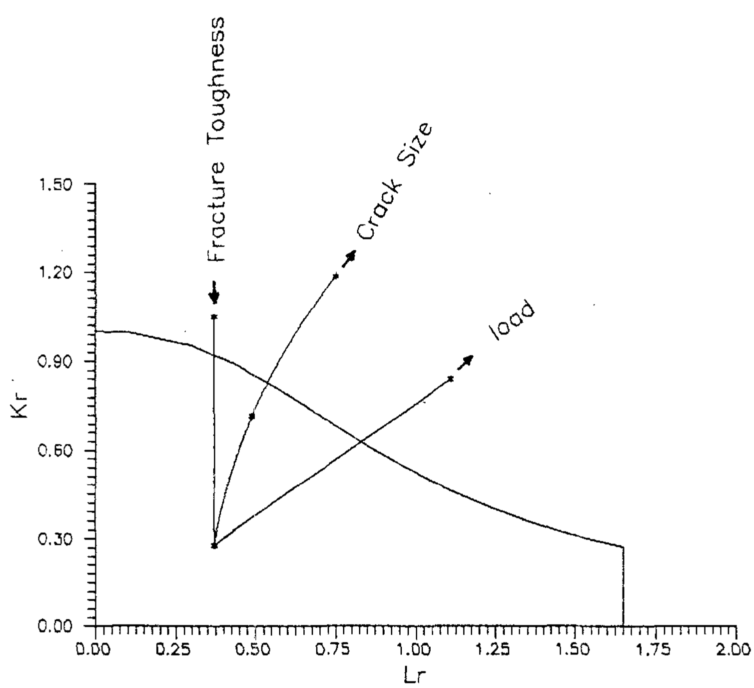

Fig. 7. Assessment points on the failure assessment diagram for various cracks, loads and fracture toughnesses of pump discharge elbow.

sensitivity study has been performed for three parameters, namely crack length, load and fracture toughness. For each of these sensitivity studies, the starting point of the locus indicates the specific case of the pump discharge elbow with a postulated circumferential crack of $2\psi_c = 90°$.

As for the sensitivity study of crack length, the circumferential crack angle is varied from 90 to 180°. Figure 7 shows the locus of the assessment points on the failure assessment diagram. The critical crack length at which a crack starts growing is $2\psi_c = 150°$. Figure 7 also shows the locus of the increasing load line. The resultant bending moment at which crack ($2\psi_c = 90°$) initiation starts is 576 kN m, which is 2·25 times higher than the maximum loading. Finally, the locus of the decreasing fracture toughness of the material is also shown in Fig. 7. The minimum fracture toughness at which a crack of dimension $2\psi_c = 90°$ starts growing is $J_{ic} = 10$ kN/m. This is near to the material fracture toughness value of $J_{ic} = 8$ kN/m at the lower-shelf temperature.[4]

6 CONCLUSIONS

The following conclusions may be drawn from the above analysis:

(1) The present work demonstrates a complete methodology for safety assessment of an elbow typical of those used in a nuclear power plant by using the R6 concept and the database suggested in Ref. 1.

(2) The case study used to compute the limit moment of the elbow by using the computer code 'FABS' shows good agreement with the results quoted in Ref. 3.

(3) The asymptotic limit moment of a non-weakened pump discharge elbow considering actual material stress/strain properties is higher than the limit moment predicted by eqn (2) using the flow stress of the material. Hence, eqn (2) may be used conservatively to compute the limit moment of a non-weakened elbow.

(4) The limit moment of a pump discharge elbow with a circumferential crack normalised with respect to the limit moment of a non-weakened elbow (eqn (2)) may be expressed by a linear relationship with crack angle. This enhances the applicability of eqn (3) in Ref. 2.

(5) With the maximum loading condition the pump discharge elbow can sustain a circumferential crack of angle $2\psi_c = 150°$ before the onset of crack growth.

(6) With a postulated circumferential crack angle of $2\psi_c = 90°$, the pump discharge elbow can sustain 2·25 times the maximum load without crack initiation.

(7) With a postulated circumferential crack angle of $2\psi_c = 90°$, the minimum permissible applied fracture toughness is $J_{ic} = 10\,kN/m$, which is near to the material fracture toughness of $J_{ic} = 8\,kN/m$ for the material at lower-shelf temperature.

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