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CONTRIBUTED Papers

# THREE-DIMENSIONAL TRANSIENT ANALYSIS OF TWO COUPLED SUBMERGED CYLINDRICAL SHELLS

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## ABSTRACT

This paper focuses attention on a three field coupled problem consisting of two cylindrical shells submerged in an acoustic medium. Method of partitioning is used successfully to partition the three fields. It is shown that the two cylinders are coupled by three-dimensional flow field and bending mode is important. The paper ends with concluding remarks for extending this method for safety analysis of submerged tubes to include non-linear fluid/structure behaviour.

KEY WORDS Transient analysis Submerged shells Dynamic behaviour

## INTRODUCTION

Dynamics of multiple submerged cylindrical shells is an important area of research which has application for nuclear industry in the area of fuel assembly design, heat exchanger and steam generator tube design. In addition underwater piles, submerged pipelines in sea and parallel tunnels are also subjected to fluid induced transient forces and their dynamic characteristics have to be understood properly for a sound and reliable design. Dynamics of multiple cylinders has been studied by Chen *et al.*[1–7] and his recent monograph[7] presents extensive results. In addition significant theoretical results are presented by Paidoussis *et al.*[8–10], Au-Yang[11], Brown[12], and Lever *et al.*[13]. The methods available are theoretical, numerical and semi-empirical to predict the complex phenomena of vortex shedding, acoustic resonance and fluid elastic excitation. The basic methodology in analysing the submerged tubes dynamics problem has been to obtain the added mass and some characteristic numbers which can predict the fluid excitation modes. The added mass is normally obtained by assuming a two-dimensional flow

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theory for such tubes, where as a three-dimensional fluid field exists for a submerged tube supported at the ends. In addition it is well known that in a compressible fluid, acousto-elastic modes may get excited resulting in significant change in the added mass. For coupled mode excitation the effect of compressibility may be important in some of the problems. In view of this it is important to develop capabilities for analysing such complex coupled problems by numerical methods, taking into account the compressibility of the fluid and the flexibility of the coupled elastic shells.

In another paper three-dimensional transient analysis of a single submerged cylindrical tube has been presented with three-dimensional finite element fluid shell interaction code FLUSHEL[14] with various support conditions applied at the ends. In that analysis the fluid has been modelled with three-dimensional brick elements, while the shell response is obtained by the two-dimensional version of nine-noded Lagrangian degenerate shell elements with explicit integration through the shell thickness. At present, the fluid model includes a compressible, inviscid fluid behaviour. The details of the element formulation and the solution scheme can be seen in another paper[15]. There it has been established that the three-dimensional response of the cylindrical shell depends on the support conditions. In the present paper some studies on two submerged cylindrical shells are presented. First, a plane strain solution is obtained for two neighbouring tubes, which are subjected to a pressure pulse from one end. Next we present the case of a pair of simply supported axially constrained tubes subjected to pressure pulse from one end. This three-dimensional transient coupled dynamics problem is very important to understand the dynamic behaviour of heat exchanger tubes, where sudden bulk pressurization may take place due to failure of any one of the neighbouring tubes. Further, we also consider the effect of local pressurization and results are given for the line wave loading. Finally, we consider the case of a line wave originating at the centre of the ligament of the two tubes. Comparison between the last two cases of the line wave loadings shows that the line wave originating at the centre of the tube causes in-phase motion of the tubes while the line wave loading from an end of one of the tubes causes out-of-phase motion of the two tubes.

## THEORY

In order to analyse multifield coupled problems, the method of partitioning is very suitable. The details of this method have been presented in another paper[15] for a two field coupled problem of three-dimensional fluid domain and shell structure. The modular nature of this method permits further extension very easily for the three field problem. We consider the fluid field $\Omega_f$ coupled to two cylindrical shells C1 and C2 as shown in *Figure 1*. The cylinder C1 interacts with fluid at $\Gamma_{s1}$ and transfers the normal surface acceleration to the fluid $\Omega_f$, similarly

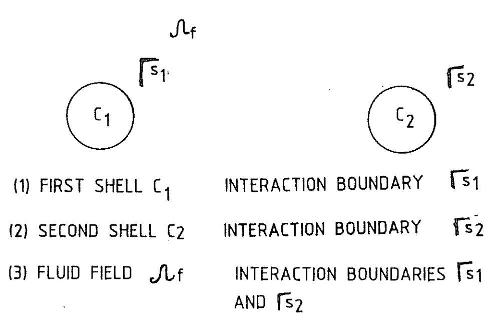


*Figure 1* Multifield coupled problem

the cylinder C2 interacts with the fluid at $\Gamma_{s2}$ and transfers the normal surface acceleration to fluid $\Omega_f$. The fluid pressure in turn is transferred from $\Omega_f$ at $\Gamma_{s1}$ and $\Gamma_{s2}$ to the cylinders C1 and C2 respectively. The dynamics of the three field problem can be presented by the second order semidiscrete ordinary differential equations of following form,

$$\begin{bmatrix} \mathbf{M}_f & -\rho_f\mathbf{Q}_{f1} & -\rho_f\mathbf{Q}_{f2} \\ \mathbf{0} & \mathbf{M}_{s1} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{M}_{s2} \end{bmatrix} \begin{bmatrix} \ddot{\mathbf{p}} \\ \ddot{\mathbf{u}}_1 \\ \ddot{\mathbf{u}}_2 \end{bmatrix} + \begin{bmatrix} \mathbf{C}_f & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{C}_{s1} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{C}_{s2} \end{bmatrix} \begin{bmatrix} \dot{\mathbf{p}} \\ \dot{\mathbf{u}}_1 \\ \dot{\mathbf{u}}_2 \end{bmatrix} +$$

$$\begin{bmatrix} \mathbf{K}_f & \mathbf{0} & \mathbf{0} \\ -\mathbf{Q}_{s1} & \mathbf{K}_{s1} & \mathbf{0} \\ -\mathbf{Q}_{s2} & \mathbf{0} & \mathbf{K}_{s2} \end{bmatrix} \begin{bmatrix} \mathbf{p} \\ \mathbf{u}_1 \\ \mathbf{u}_2 \end{bmatrix} = \begin{bmatrix} \rho_f\mathbf{Q}_{f1}\ddot{\mathbf{u}}_{g1} + \rho_f\mathbf{Q}_{f2}\ddot{\mathbf{u}}_{g2} + \mathbf{f}_f \\ -\mathbf{M}_{s1}\ddot{\mathbf{u}}_{g1} + \mathbf{f}_{s1} \\ -\mathbf{M}_{s2}\ddot{\mathbf{u}}_{g2} + \mathbf{f}_{s2} \end{bmatrix} \quad (1)$$

In the above equation $\mathbf{M}_f$, $\mathbf{C}_f$ and $\mathbf{K}_f$ are the fluid mass, damping and stiffness matrices respectively, $\mathbf{Q}_{f1}$ and $\mathbf{Q}_{f2}$ are the coupling terms to transfer the acceleration data from C1 and C2 structures to the fluid domain. $\rho_f$ is the fluid density, $\ddot{\mathbf{u}}_{g1}$ and $\ddot{\mathbf{u}}_{g2}$ are the ground motions of the two structures, $\mathbf{f}_f$ is the applied fluid force and $\mathbf{p}$ denotes the pressure field. A superposed dot denotes differentiation with respect to time. $\mathbf{M}_{si}$, $\mathbf{C}_{si}$ and $\mathbf{K}_{si}$ $(i = 1, 2)$ denotes the mass, damping and stiffness respectively of the $i$th structures. $\mathbf{Q}_{si}$ $(i = 1, 2)$ is the coupling matrix to transfer the fluid pressure to the shell surface, $\mathbf{u}_i$ $(i = 1, 2)$ is the displacement vector and $\mathbf{f}_i$ $(i = 1, 2)$ is the specified structure force vector.

In the present problem the solution scheme is similar to that given in another paper[15]; however, to save the computer time and storage space, identical mesh is chosen for the two cylinders, thus all the structural matrices have to be evaluated only once if both the cylinders are identical in all respects. Different node numbers are assigned to the two cylinders only for recognizing the field variables alone. This results in significant saving in terms of storage space as each shell node has five degrees of freedom which otherwise may result in to large size of coefficient matrices. The advantages of the method of partitioning may be recognized here. Although (1) is fully populated, sequential treatment of the individual fields (or parallel treatment if parallel processors are available) makes this method very powerful to treat the coupled multifield problem. In this case also either an explicit or an implicit integrator can be selected for various fields depending on the critical time periods of each mesh as discussed in another paper[15].

## CASE STUDIES

Now we present the transient analysis of two identical submerged cylindrical tubes. The pair of cylinders are identical to the single cylinder problem presented in another paper[15].

### *Two submerged cylindrical tubes with a specified pressure history from one end*

*Figure 2* shows two identical submerged cylindrical tubes of radius 11.82 in, thickness 0.12 in, which are separated with a centre-to-centre distance of 30 in. This problem has been analysed by Geers and Ruzicka[16,17] with plane strain assumptions, where it is assumed that a step pressure pulse impinges on the first cylinder C1. This is an important structure–fluid–structure interaction problem, where three fields are involved. The motion of C1 cylinder excites the surrounding fluid which in turn excites C2 cylinder. Both the cylinders are assumed to be submerged in an infinite acoustic medium. Two-dimensional analysis of single submerged cylindrical shell has been reported by Geers *et al.*[18], for which exact analytical results are available. In another

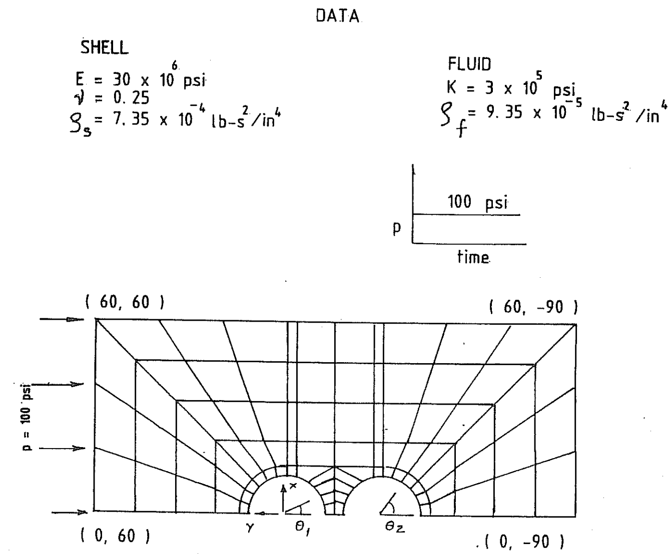


*Figure 2* Two submerged cylindrical shells in infinite fluid

paper[15], it has been shown that the three-dimensional response of a submerged tube is quite different than the simple two-dimensional plane strain response due to inclusion of bending mode along with the resulting three-dimensional fluid response. Another important consideration in the analysis of submerged tubes is the type of wave loading. Two types of loads are considered, one due to a uniform surface wave and another due to a line wave. These two types of waves typically represent the bulk pressurization of the acoustic medium and the local pressurization due to some accident conditions.

*Plane strain case of a pair of tubes.* In the two submerged cylinders model of *Figure 2* two layers of three-dimensional trilinear fluid elements have been considered. The cylinders are modelled with nine-noded degenerate shell elements with $2 \times 12$ mesh for each cylinder. Half lengths of the cylinders (60 in) have been considered for the analysis, with symmetry condition at the mid span. A uniform pressure pulse of 100 lb/in$^2$ is applied at the left end while at $X = 60$ in and $Y = -90$ in radiation boundary condition is applied. First, we consider the two-dimensional plane strain case (case PS) for which both the edges of the two cylinders are adequately constrained. *Figure 3a* shows the radial velocity response $\rho_f c\dot{w}/PI$ of both the cylinders (C1 and C2) at the front edge of quarter span ($\theta_{1,2} = \Pi, L/4$), and at the rear edge of quarter span ($\theta_{1,2} = 0, L/4$). In the velocity parameter, $\rho_f$ is the fluid density, $c$ is the acoustic speed, $\dot{w}$ is the radial velocity and $PI$ is the pressure pulse intensity (100 lb/in$^2$ in this case). In this Figure, the response of single submerged cylinder in plane strain condition is also included for comparison purpose, the analysis of which has been presented in another paper[15]. Due to plane strain condition the responses at the quarter and mid spans are identical in both the cases. In cylinder C1 the front edge ($\theta_1 = \Pi$) responds in a manner similar to the single submerged cylinder case, while the rear edge ($\theta_1 = 0$) shows a higher response compared to the single cylinder case. The response at the front edge ($\theta_2 = \Pi$) of C2 cylinder is significantly higher than the single submerged cylinder response. At the rear edge of C2 cylinder initially the velocity response is low, but it approaches the plane strain value as the wave reaches the rear shell C2. This time lag is also noted in the case of response at the front edge of C2 cylinder. Once the

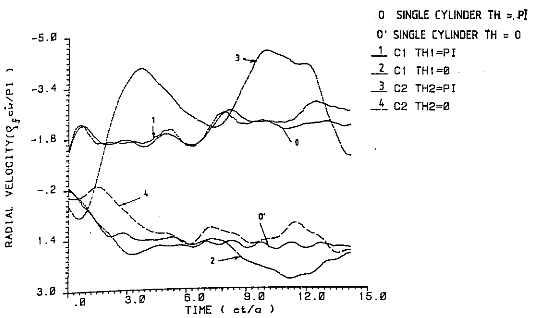


*Figure 3(a)* Two submerged cylinders in infinite fluid (PS)

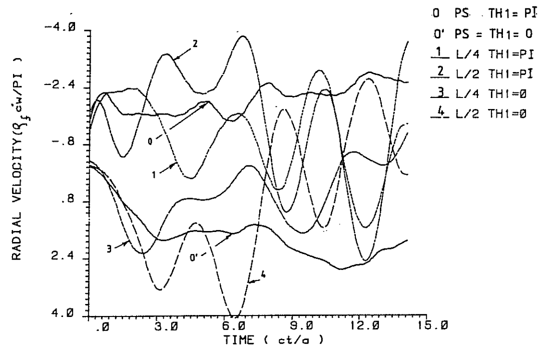


*Figure 3(b)* Two submerged cylinders in infinite fluid C1 (SSASUW)

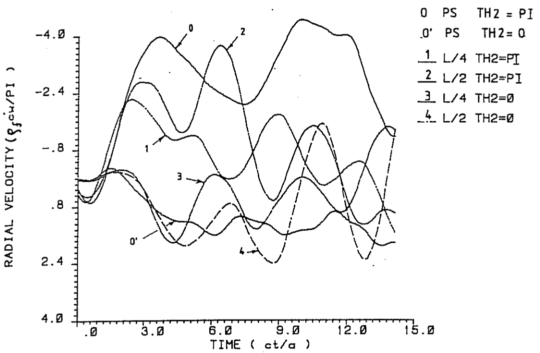


*Figure 3(c)* Two submerged cylinders in infinite fluid C2 (SSASUW)

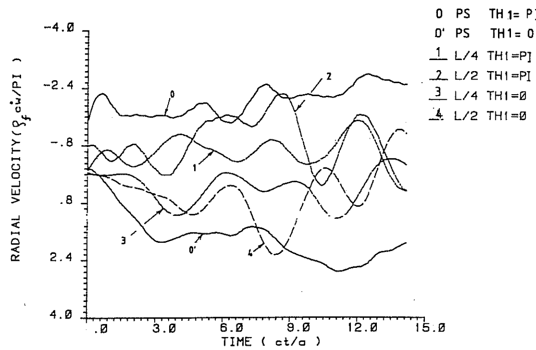


*Figure 3(d)* Two submerged cylinders in infinite fluid C1 (SSALIW)

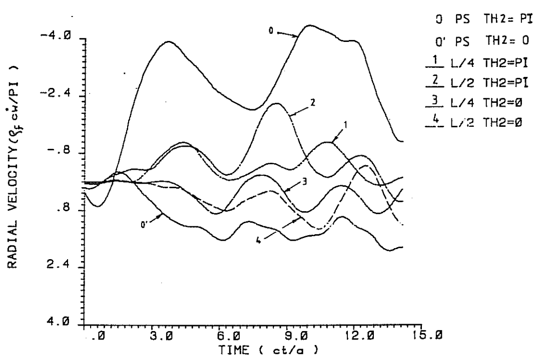


*Figure 3(e)* Two submerged cylinders in infinite fluid C2 (SSALIW)



scattered wave reaches from C1 cylinder to C2 cylinder the velocity response increases significantly. Similar results have been reported by Geers and Ruzicka[16,17] for plane strain case.

*Three-dimensional response of the tubes.* The other cases of interest are of two simply supported submerged cylindrical tubes excited with a surface wave through the full span (case SSASUW) to consider the case of bulk pressurization and the second of a pair of simply supported tubes excited with a line wave (case SSALIW) at the centre. The latter is useful to understand the response in the case of local pressurization. In both the cases the tubes are assumed to be axially constrained. Similar studies have been reported in another paper[15] for a single submerged tube. In the present case again both the tubes have the fundamental frequency of 86.4 Hz in the first axial ($m = 1$) and the third circumferential ($n = 3$) mode. *Figures 3b* and *3c* present the velocity responses of C1 and C2 cylinders respectively for case SSASUW along with the results of two submerged cylinders for plane strain case (PS) for comparison purpose. Significant increase in velocity response is apparent for C1 cylinder compared to the two-dimensional PS response. The velocity response of C2 cylinder is also seen to be higher than the two cylinder plane strain solution (PS) at $\theta_2 = 0$ (*Figure 3c*); however, it is less significant than the case of C1 cylinder (at $\theta_1 = 0$) presented in *Figure 3b*. This is due to the effect of wave scattering from C2 cylinder which increases the velocity response in C1 cylinder at the rear edge ($\theta_1 = 0$) significantly. Effect of wave scattering is also recognized at the front edge of C2 cylinder; however, to a lesser extent due to loss in energy of the wave coming from C1 to C2 and smaller pressure difference between the front edge and the rear edge of C2 cylinder compared to that of C1 cylinder as discussed in the next paragraph. At the rear edge of C2 cylinder ($\theta_2 = 0$) again time lag is noticed, but the response builds up to higher value compared to the PS case. For both of the cylinders the mid span velocities are higher than the quarter span values. Next we consider the case of a pair of submerged tubes loaded with a line wave at the centre (case SSALIW). *Figures 3d* and *3e* show the velocity response of the two tubes at the above mentioned locations. In this case the net load reduces so the response is seen to be lower than the SSASUW case. Here also the mid span velocity response is seen to be higher than the quarter span velocity response and the effect of wave scattering is again noticed at the rear edge ($\theta_1 = 0$) of C1 cylinder and the front edge ($\theta_2 = \Pi$) of C2 cylinder.

Now we present the pressure response of both the cylinders at the front and rear edges. *Figures 4a–4d* show the pressure response at the quarter and mid spans for cases PS, SSASUW and SSALIW. It is noticed that for the case SSASUW the front edge pressure is more significant than the rear edge pressure for C1 cylinder and the mid span pressure is higher than the quarter span pressure for both the edges. Similar behaviour is noticed for case SSALIW also and the pressure response is significantly higher than the PS case. In case of C2 cylinder the pressure response increases significantly at the front edge due to scattering and again the mid span response is higher than the quarter span response. In this case also the line wave loading results into a higher pressure response compared to the two-dimensional case PS. The pressure response is consistent with the velocity response of the cylinders. Although the pressure response at the front edge of C2 cylinder is slightly higher than the front edge of C1 cylinder due to multiple scattering; however, the back pressure at the rear edge for C1 cylinder is quite less than that for C2 cylinder. This clearly explains the higher velocity response in case of C1 cylinder compared to C2 cylinder described in the previous paragraph.

*Two submerged cylindrical tubes with a specified pressure history at the centre of the ligament*

The other important coupled three field problem of shell fluid interaction is of two submerged cylindrical tubes which are loaded with a line wave originating at the centre of their ligament. This problem may arise in the case of heat exchanger tubes, where the concern is to study the effect of sudden pressurization of the acoustic medium, resulting from failure and subsequent release of high pressure fluid from one tube, on the neighbouring tubes. The mesh of *Figure 2* is again used for this problem, with a specified line wave of 100 lb/in$^2$ in between the ligaments as shown in *Figure 5*. The boundaries at $Y = -90$ in and $X = 60$ in are assumed to be of radiating type, while the boundary at $Y = 60$ in is assumed to be fixed. Again the tubes are

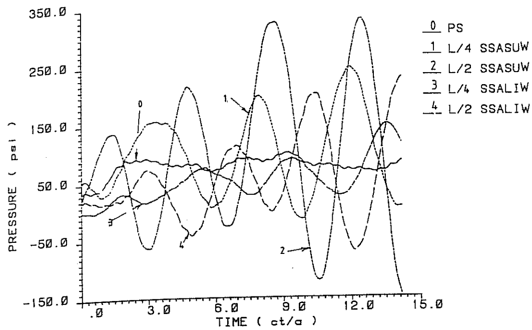


*Figure 4(a)* Pressure history (theta1 = Π) C1 cylinder

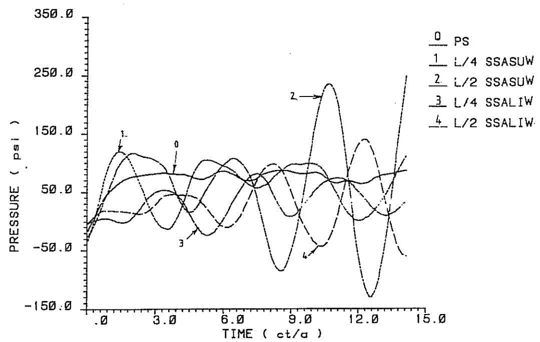


*Figure 4(b)* Pressure history (theta1 = 0) C1 cylinder

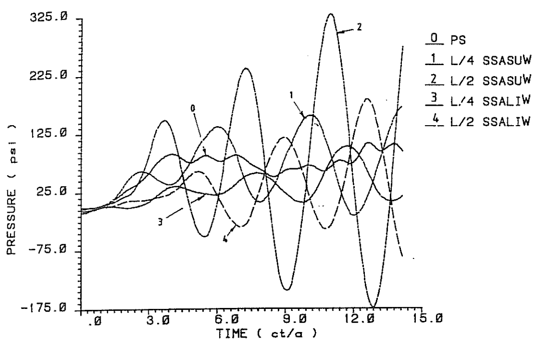


*Figure 4(c)* Pressure history (theta2 = Π) C2 cylinder

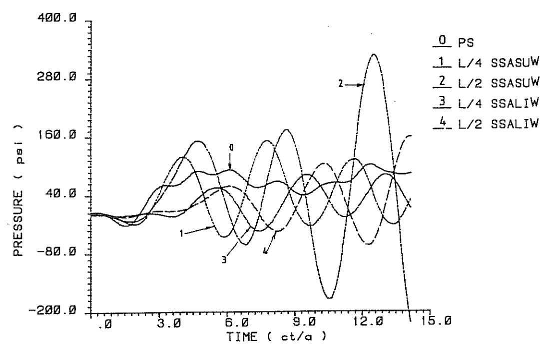


*Figure 4(d)* Pressure history (theta2 = 0) C2 cylinder

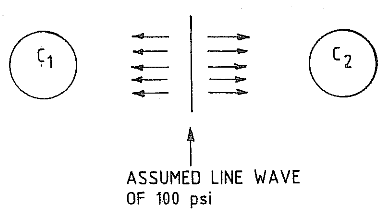


*Figure 5* Two cylinder problem with line wave at centre of ligament

assumed to be simply supported with axial restraint conditions at the ends. The length of the line wave in the present model is assumed to be of 11.82 in, which is equivalent to the assumption of complete failure of any one of the neighbouring tubes leading to an impulsive line wave. *Figures 6a* and *6b* show the radial velocity response of C1 and C2 cylinders on the front and rear edges at the quarter span along with the mid span. It is noticed that at the rear edge of C1 cylinder and the front edge of C2 cylinder the velocities are the maximum due to multiple scattering. The mid span velocity is more than the quarter span velocity for both the cylinders. The pressure response at the above locations are shown in *Figures 7a* and *7b* for C1 and C2 cylinders respectively. It is noted that the maximum response is at the mid spans of the rear edge of C1 cylinder and the front edge of C2 cylinder which is consistent with the velocity response of the two tubes. *Figures 8a* and *8b* show the axial membrane force ($N_{xx}$) for the

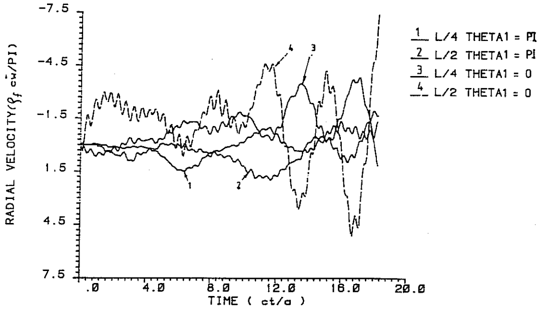


*Figure 6(a)* Velocity response in C1 cylinder (fix–rad)

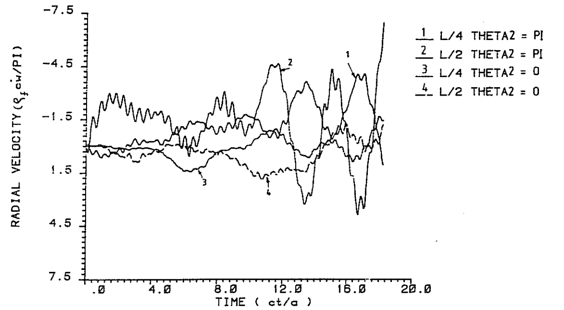


*Figure 6(b)* Velocity response in C2 cylinder (fix–rad)

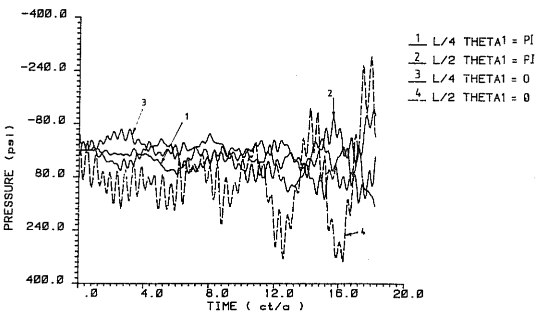


*Figure 7(a)* Pressure response in C1 cylinder (fix–rad)

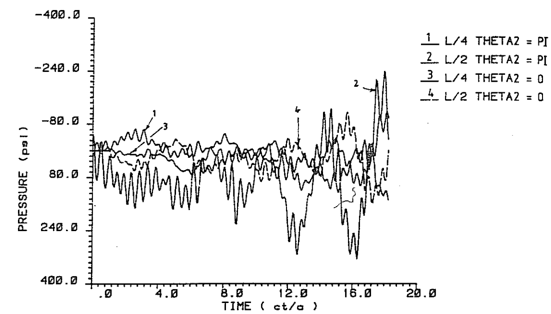


*Figure 7(b)* Pressure response in C2 cylinder (fix–rad)

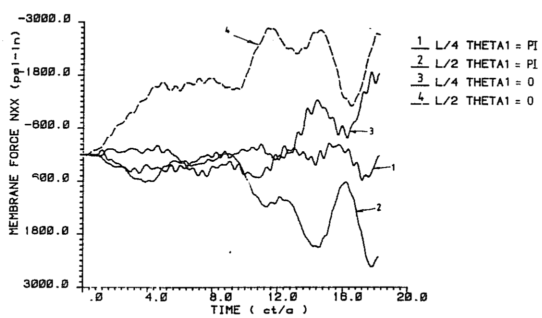


*Figure 8(a)* Membrane force $N_{xx}$ in C1 cylinder (fix–rad)

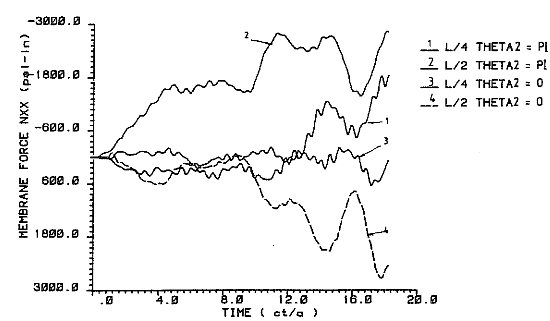


*Figure 8(b)* Membrane force $N_{xx}$ in C2 cylinder (fix–rad)

two cylinders respectively. Again the mid span response is found to be more than the quarter span response and the rear edge of C1 cylinder along with the front edge of C2 cylinder show responses higher than their respective opposite faces due to wave scattering effects. The circumferential membrane force ($N_{yy}$) response is presented in *Figures 9a* and *9b* for cylinders C1 and C2 respectively. It is observed that circumferential membrane force ($N_{yy}$) is higher than axial membrane force and effect of multiple scattering of waves is noticed at the opposite faces of cylinders facing each other. *Figures 10a* and *10b* show the axial moment ($M_{xx}$) and *Figures*

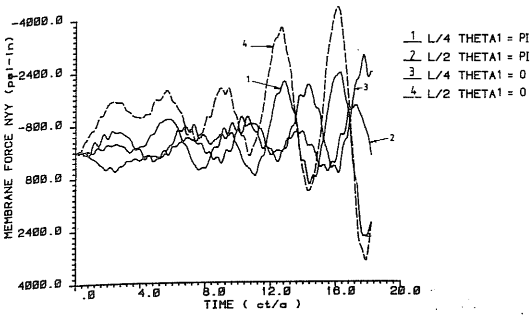


*Figure 9(a)* Membrane force $N_{yy}$ in C1 cylinder (fix–rad)

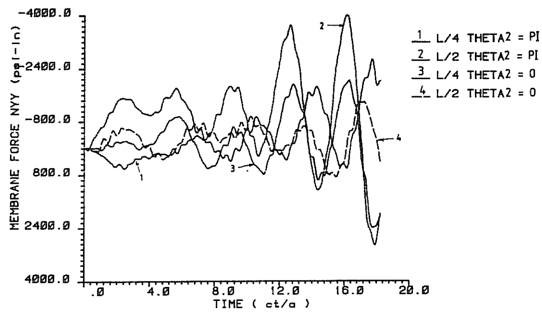


*Figure 9(b)* Membrane force $N_{yy}$ in C2 cylinder (fix–rad)

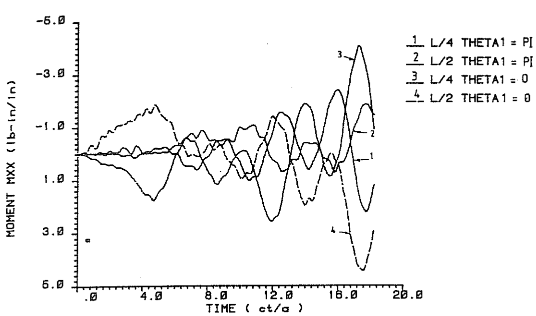


*Figure 10(a)* Moment $M_{xx}$ in C1 cylinder (fix–rad)

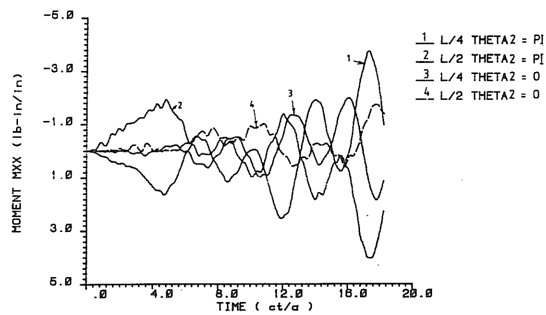


*Figure 10(b)* Moment $M_{xx}$ in C2 cylinder (fix–rad)

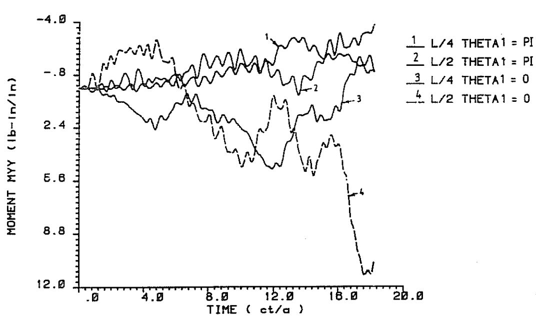


*Figure 11(a)* Moment $M_{yy}$ in C1 cylinder (fix–rad)

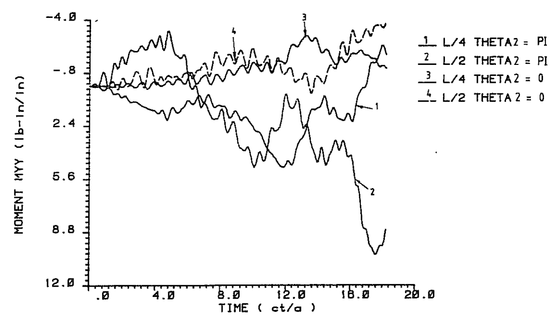


*Figure 11(b)* Moment $M_{yy}$ in C2 cylinder (fix–rad)

*11a* and *11b* show the circumferential moment ($M_{yy}$) at the earlier mentioned locations of the tubes. It is observed that the circumferential moment is significant than the axial moment, and the response is higher at the mid span compared to the quarter span. In these cases also the effect of scattering may be noticed at the rear edge ($\theta_1 = 0$) of C1 cylinder and the front edge ($\theta_2 = \Pi$) of C2 cylinder. Another important observation is that the response is more in case of C1 cylinder compared to C2 cylinder. This is due to the wave reflection from the boundary ($Y = 60$ in) opposite to C1 cylinder.

## CONCLUSIONS

Coupled transient analysis of multiple field problems can be efficiently carried out by the method of partitioning. The example problems reported in this paper demonstrate that the bending mode is important for two neighbouring submerged cylinders excited either by a surface wave or a line wave. The analysis shows that the fluid field is three-dimensional in nature and simplified two-dimensional plane strain analysis need not be conservative for such coupled submerged tubes. Similar conclusions were arrived at in another paper[15] for the single submerged cylindrical shell fluid–interaction problem. Analysis has been presented for two simply supported cylinders excited by a surface wave and a line wave. In this case the rear edge of C1 cylinder and the front edge of C2 cylinder oscillate in out-of-phase motion. Another case is considered where the pressure pulse is assumed to originate in between the ligament of the two cylinders. In this case the rear edge of C1 cylinder and the front edge of C2 cylinder oscillate in the same phase. Therefore the velocity response for this case is more than the former case as seen by comparing *Figures 3d* and *3e* with *Figures 6a* and *6b* respectively. The methodology of transient analysis presented in this paper can be used to assess the dynamic behaviour of coupled submerged tubes used in heat exchangers and underwater pipe lines. Method of partitioning can be used to analyse complex fluid multi-structure interaction problems which include non-linear behaviour of the fluid and/or the multiple structures.

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