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ON THE USE OF ONE POINT AND TWO POINTS SINGULARITY ELEMENTS IN THE ANALYSIS OF KINKED CRACKS

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SUMMARY

A 3-noded triangular two singular points finite element is proposed to model two variable order singularities lying at close proximities as in the case of a kinked crack. The element meets the rigid body mode and the interelement compatibility of the convergence criteria. A degenerate form of the element is a one point variable order singularity element, which meets all the convergence requirements. A number of kinked crack problems have been analysed using the new element and its degenerate form. The computed results are compared with analytical solutions. The accuracy of the results is found to be good.

INTRODUCTION

Analysis of kinked cracks is complicated by the existence of two singular points separated by a small distance. In most of the cases one of the singularities, which occurs at the knee, is of a variable order and the other, at the crack tip, is a square-root singularity. We can analyse such a problem by using quarter point elements at the crack tip and the one point variable order singular elements suggested by Tracey and Cook[1] at the knee. We have demonstrated recently[2] that the use of two points singularity elements instead is very advantageous. It leads to both computational gains and accuracy. We have presented a 4-noded element for this purpose. This element does not meet any of the three convergence requirements—the rigid body mode, the constant strain condition and the interelement compatibility.

We present here a 3-noded triangular element which meets both the rigid body mode and the compatibility conditions, when it is used as a two points singularity element. We also show that the element can be used as a one point variable order singularity element like the Tracey and Cook element.[1] This element then meets all the three convergence requirements and hence it does not suffer from the limitation of the Tracey and Cook element.[1]

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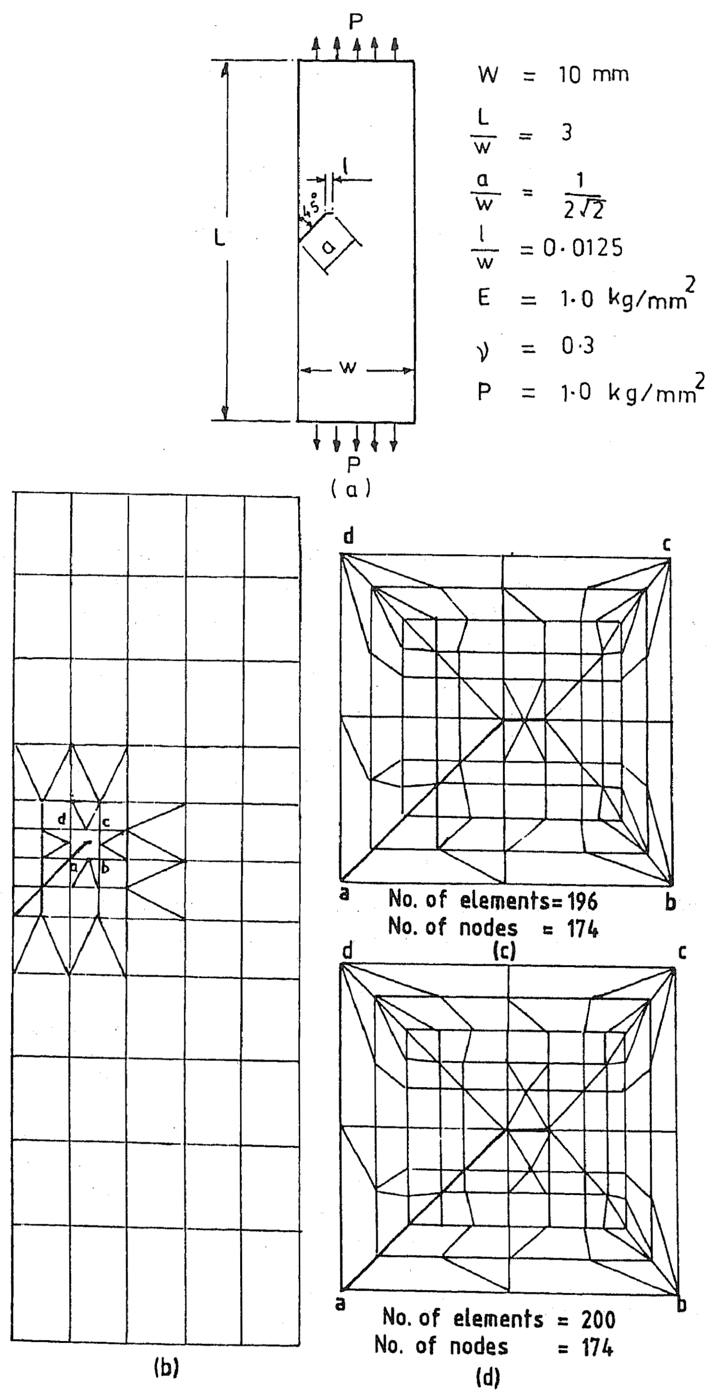

Figure 3. (a) Single edge kinked crack in a long tension strip. (b) Discretization details away from the knee and crack tip. (c) and (d) Two discretization schemes around the knee and crack tip

cases. A 9-point integration scheme has been used for the quadrilaterals and a 7-point scheme is employed for the triangular elements.

The first example deals with a kinked crack in a long elastic tension strip. This problem was studied by Tracey and Cook[1] as well as by us in Reference 2. The dimensional details are shown in Figure 3(a). In the present study two discretization schemes around the knee and crack tip (Figures 3(c) and 3(d)) were used. Through the first scheme the performances of one point singularity elements of the Tracey and Cook[1] type and degenerate TSPT elements have been examined separately. In the second scheme one TSPT element surrounded by a number of degenerate TSPT elements has been used. The following equations were used to calculate stress intensity factors by comparing the net opening and sliding displacements, as suggested in Reference 5.

$$
\begin{aligned}
K_{\mathrm{I}} &= (E\delta_{\mathrm{v}}\sqrt{2\pi/r})/8 \\
K_{\mathrm{II}} &= (E\delta_{\mathrm{u}}\sqrt{2\pi/r})/8
\end{aligned}
\tag{10}
$$

where δ_{v} and δ_{u} are the net opening and sliding displcements at the knee respectively, r is the kink length and E is the modulus of elasticity. These results are shown in Table I. This table also includes the result quoted in Reference 1, wherein apparently a very large number of elements have been employed near the crack tip.

The second example is the analysis of a double edge kinked crack in a long tension strip, as shown in Figure 4(a). The kink angle was varied from 15° to 90° in steps of 15°. Figure 4(b) shows the discretization details away from the kink. The arrangement of elements around the crack tip and the knee for the case of 45° is shown in Figure 4(c). The co-ordinates of the nodes around the crack tip and the knee were suitably modified to obtain discretization for the cases $\theta = 90°$, 75°, 60°, 30° and 15°. The stress intensity factors were computed as before by comparing the displacements of the nodes at the knee using equation (10). The J-integral was computed using three different contours around the crack tip lying within C_1 and C_2 (Figure 4(c)). Table II shows a comparison of computed J-integrals with the results obtained using the approach of Reference 6. This table also shows the order of singularities at the knee for different kink angles, which were calculated using Reference 3. The variation of stress intensity factor (SIF) with the kink angle is shown in Figure 5, where results based on the first order analytical solution[6] are also included.

The third example is the analysis of a single edge crack under 4-point bending. The study is again done for θ in the range 15° to 90° in steps of 15°. The mesh arrangement is similar to the discretization schemes shown in Figure 4. The stress intensity factors are again calculated by

Table I. Computed values of mode-I SIF for single edge kinked crack in a long tension strip

	Computed stress intensity factor using		
	One singular point elements (Figure 3(c))		
Computed by Tracey & Cook (Ref. 1)	Proposed by Tracey & Cook (Ref. 1)	Proposed in the present work	Two singular points element (Figure 3(d))
2·317	2·175	2·201	2·327

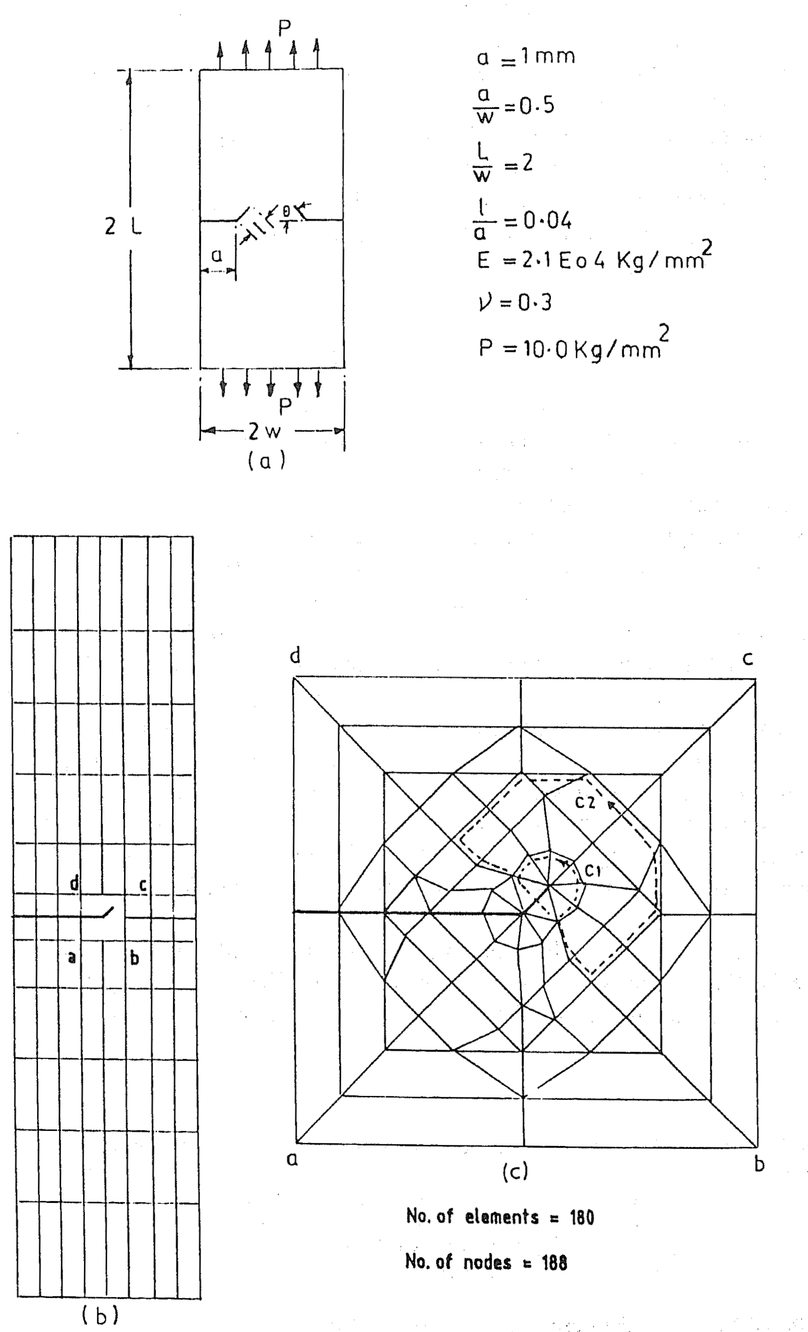

Figure 4. (a) Double edge kinked cracks in a tension strip. (b) and (c) Discretization details for $\theta = 45°$

Table II. Comparison of computed and analytical J's for double edge kinked crack

			J compound		
θ	λ at knee	J analytical	J based on three contours	Average J	Percentage difference
15°	0·8573	0·019858	0·020994 0·021688 0·021419	0·021367	+ 7·6%
30°	0·7520	0·017890	0·018369 0·018379 0·018213	0·018320	+ 2·4%
45°	0·6736	0·014970	0·015215 0·015507 0·015569	0·015430	+ 3·1%
60°	0·6157	0·011560	0·011397 0·011374 0·011424	0·011398	− 1·4%
75°	0·5738	0·008140	0·008719 0·008927 0·008710	0·008785	+ 7·9%
90°	0·5445	0·005138	0·005752 0·005445 0·005886	0·005694	+ 10.8%

comparing the displacements of the nodes at the knee. They are compared with the analytical solution[6] in Figure 6.

The last example deals with an asymmetrically branched crack in an infinite plate subjected to a unidirectional tension load, as shown in Figure 7. Only the branch angles 15° and 45° were examined here. For the proportion shown in Figure 7 the plate can very well be treated as an infinite body. The discretization scheme employed is shown in Figure 7. In the analysis the kink dimension was held constant but the main crack was varied using a scheme similar to that mentioned in Reference 2. The J-integral was computed along three different contours. The results are shown in Table III. A comparison of J is shown with the analytical results of References 7 and 8 in Figures 8 and 9 for branch angles 15° and 45° respectively. Stress intensity factors quoted in References 7 and 8 were converted in terms of J for this comparison.

DISCUSSION AND CONCLUSION

The 4-noded element of Reference 2 indicated in a sense the feasibility of having two singular points in a single element. The element failed to satisfy any of the convergence criteria. The present element has helped us to move a greater step forward. The element satisfies the two important convergence criteria—the rigid body mode and the interelement compatibility. The non-satisfaction of the third criterion, the constant strain condition, renders the element, at the most, unsuitable for the analysis of thermal problems. The element has been used to solve different shape and sizes of kinked cracks to show its accuracy.

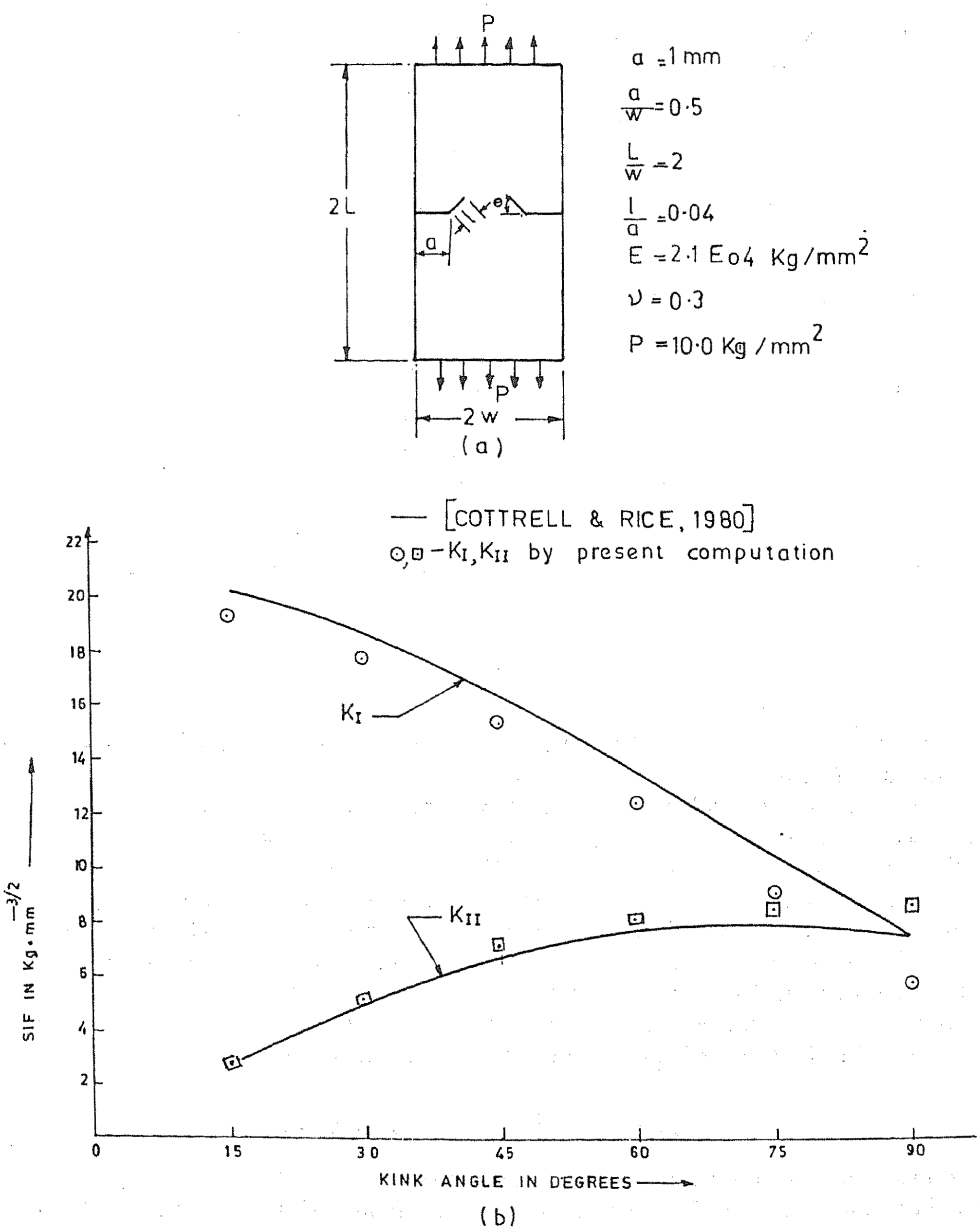

Figure 5. (a) Double edge kinked cracks in a tension strip. (b) Comparison of computed SIFs with an analytical solution

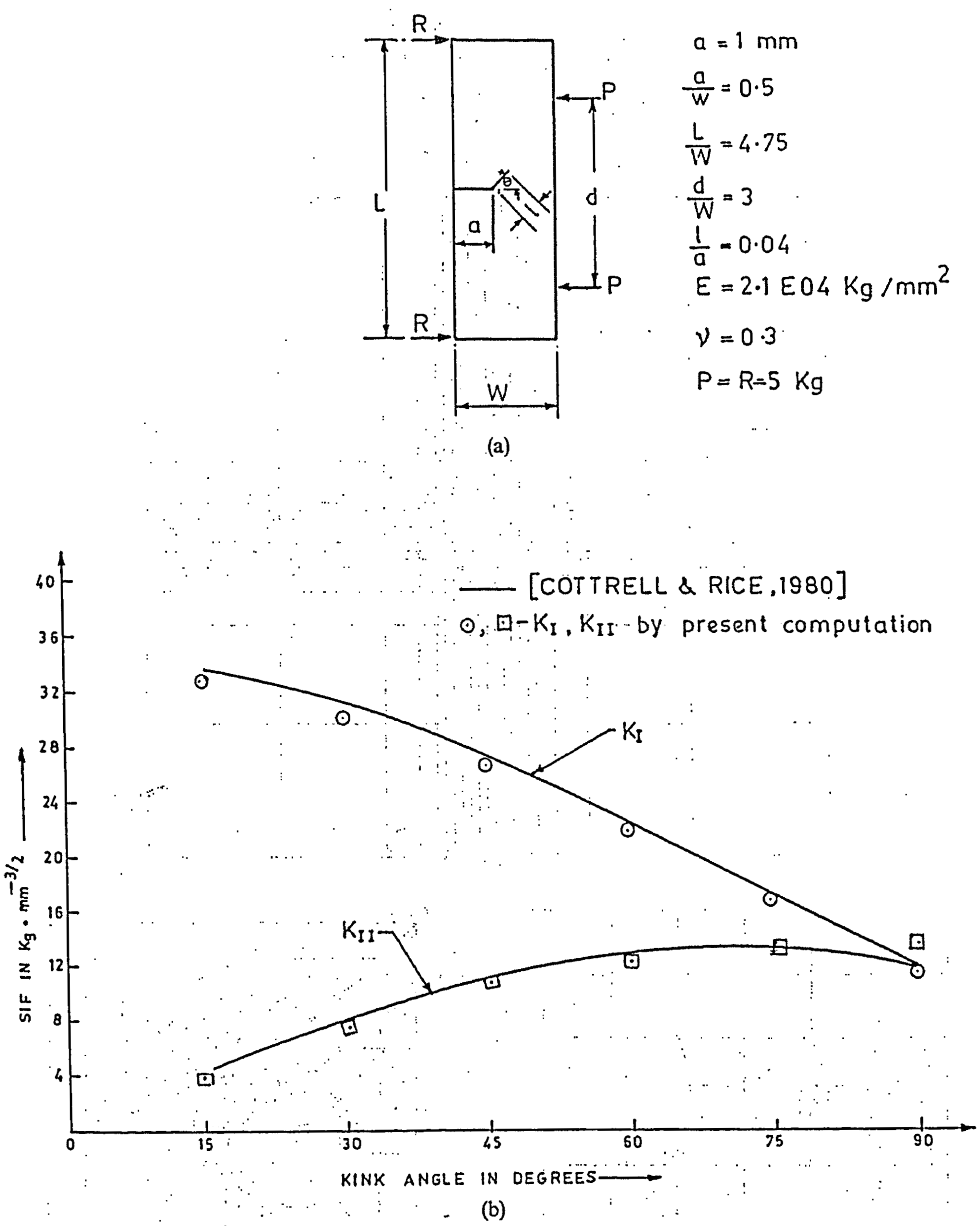

Figure 6. (a) Single edge kinked crack under four point bending. (b) Comparison of computed SIFs with an analytical solution

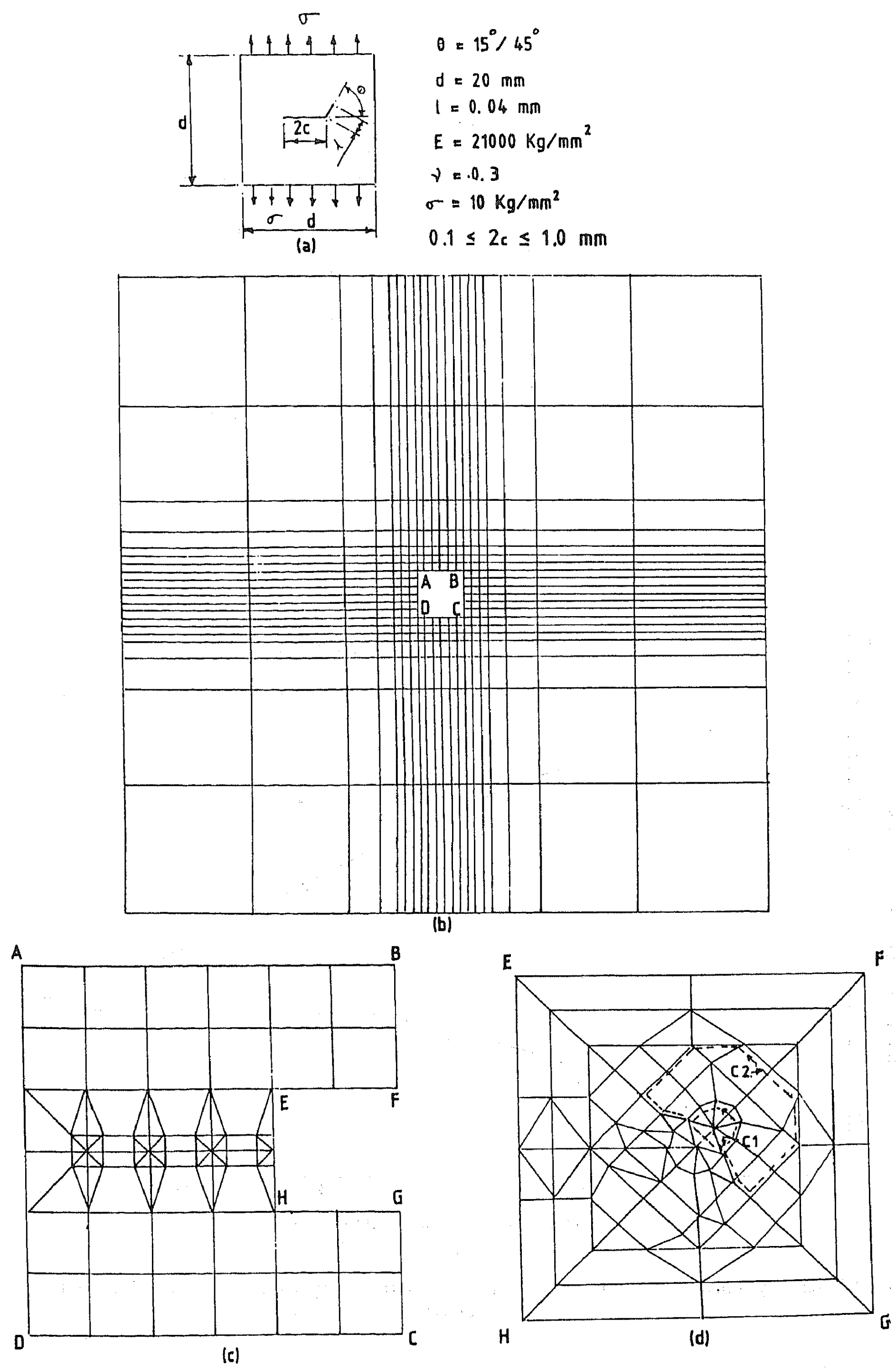

Figure 7. (a) Asymmetrically branched crack in an infinite plate under tension. (b), (c) and (d) Discretization schemes near to and away from the crack

Table III. Computed J's for asymmetric branched crack in an infinite plate

c	c/l	Computed J for kink angle 15°: J based on 3 contours	Average J	$\frac{JE}{\sigma^2 \pi c}$	Computed J for kink angle 45°: J based on 3 contours	Average J	$\frac{JE}{\sigma^2 \pi c}$
0·5	12·50	0·0074285 0·0078739 0·0078136	0·007705	1·030	0·0052993 0·0053810 0·0054151	0·005365	0·7172
0·375	9·375	0·0055719 0·0059244 0·0058875	0·005794	1·033	0·0039546 0·0040426 0·0040870	0·004028	0·7180
0·250	6.250	0·0039360 0·0039740 0·0039576	0·003956	1·056	0·0027162 0·0026579 0·0026831	0·002686	0·7181
0·125	3·125	0·0020544 0·0020887 0·0020859	0·002076	1·110	0·0013786 0·0013918 0·0014063	0·001392	0·7445
0·075	1·875	0·0013559 0·0013850 0·0013840	0·001375	1·167	0·0009154 0·0009181 0·0009192	0·000917	0·8178
0·05	1·25	0·0010037 0·0010053 0·0010050	0·001005	1·343	0·0006577 0·0006630 0·0006571	0·000657	0·8792

In the first example, a kinked crack in a long tension strip, the accuracies obtained by the Tracey and Cook[1] element and the degenerate TSPT elements are close. This example also shows that a better accuracy is obtainable if one TSPT is employed instead of two one singular point elements.

In the second example, a double edge kinked crack in a long tension strip, the SIFs agree closely with the analytical solution[6] for kink angles less than 60°. The difference is more for higher kink angles. For the 90° case, we observe that K_I is less than the analytical value and K_{II} is more than the analytical result. A similar agreement is also found for the J-integral. We have similar observations for example 3, but in this case the agreement between analytical and computed SIFs is better. In the last example, the asymmetrically branched crack in an infinite plate, the average J-integral is found to be nearer (within ± 5 per cent) to the values quoted in Reference 7 than those of Reference 8.

These case studies show that the one point singular element, i.e. degenerate TSPT element, given by equation (9) is as accurate as the Tracey and Cook[1] element. The present element is preferable to the Tracey and Cook element because it meets all the convergence requirements. Similarly, the two singular points triangular (TSPT) element represented by equation (6) can be recommended over the 4-noded two singular points quadrilateral (TSPQ) element given in Reference 2 because of its ability to meet some of the convergence criteria. Table IV shows a comparative study of the abilities to satisfy various convergence criteria by the elements considered in this paper.

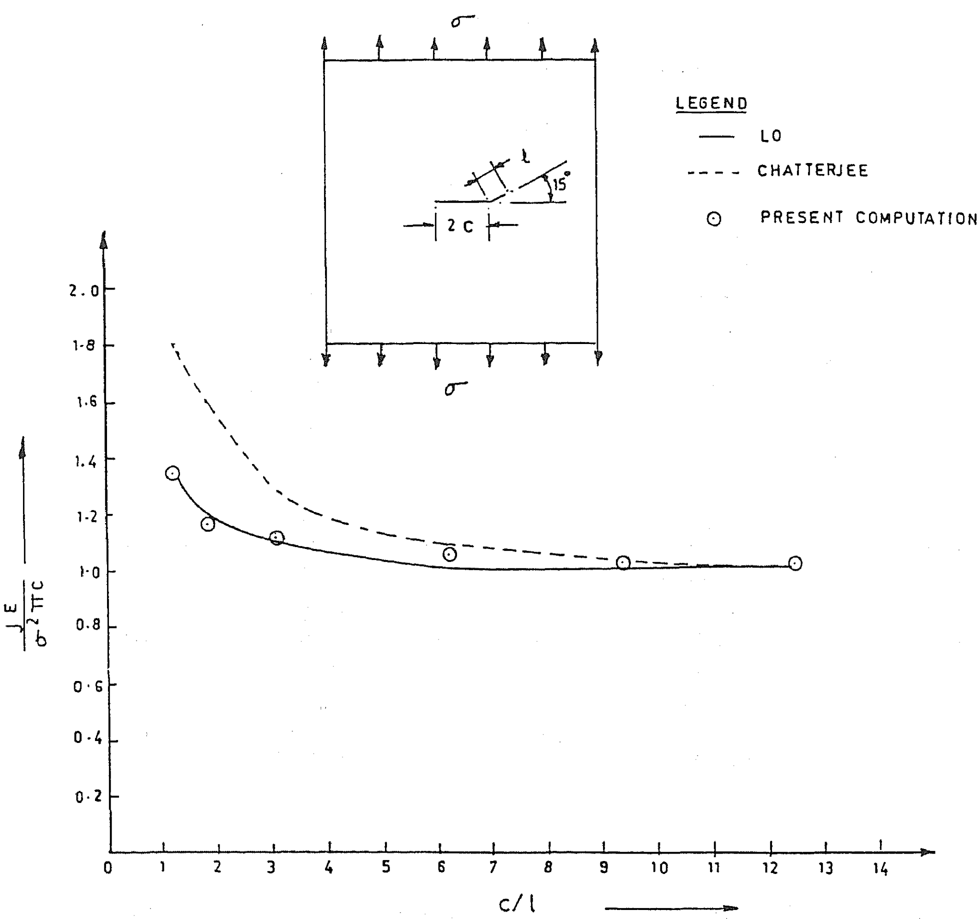

Figure 8. Comparison of analytical and computed J for the case of 15° asymmetrically branched crack in an infinite plate

Table IV. A comparative study of the abilities to satisfy convergence criteria by different elements considered

	One singular point element		Two singular points element	
Convergence criteria	Tracey & Cook element (Ref. 1)	Degenerate TSPT element	TSPQ element (Ref. 2)	TSPT element
Rigid body mode	Satisfies	Satisfies	Does not satisfy	Satisfies
Constant strain criteria	Does not satisfy	Satisfies	Does not satisfy	Does not satisfy
Interelement compatibility	Satisfies	Satisfies	Does not satisfy	Satisfies

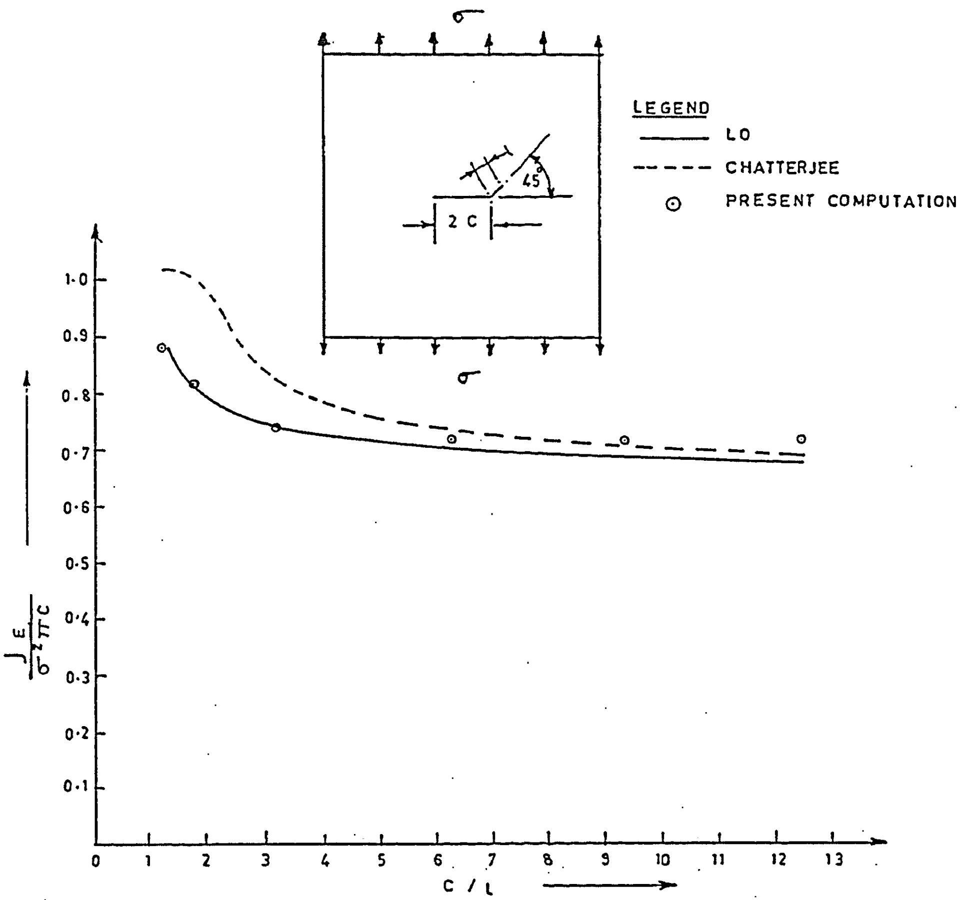

Figure 9. Comparison of analytical and computed J for the case of 45° asymmetrically branched crack in an infinite plate.

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